FlatPPL 0.1 / GAMMA-RAY ASTRONOMY

Forward-folding a photon spectrum

EXPECTED SOURCE COUNTS

$$s_r = \sum_j M_{rj} T A_j w_j \varphi_j$$

COMPLETE EXAMPLE

Three true energies, two detector bins

```
%%%
One flux amplitude, three true energies.
%%%
flatppl_compat = "0.1"
E = [0.5, 1.0, 2.0]
width = [0.5, 0.75, 1.0]
area = [100000.0, 200000.0, 300000.0]
tau = [0.05, 0.2, 0.5]
time = 10000.0

A M = rowstack([[0.7, 0.2, 0.0],
                [0.1, 0.6, 0.7]])

B unit_flux = E .^ (-2) .* exp.(-0.8 .* tau)
fold(a) = M * (time .* area .* width
    .* (a .* unit_flux))
amplitude = 0.00000001
signal = fold(amplitude)
a = elementof(nonnegreals)
bg = [2.0, 3.0]

C on ~ Poisson.(fold(a) .+ bg)
off ~ Poisson.(4.0 .* bg)

D data = record(on = [7, 8], off = [8, 12])
K = kernelof(record(on = on, off = off), a = a)
L = likelihoodof(K, data)
score = logdensityof(L, record(a = amplitude))
```

Expected counts before and after migration

```
kept = aggregate(sum, [.j], M[.r, .j])
lost = 1 .- kept
```

Before migration the three energies contribute 37.0253 expected counts. The two reconstructed bins retain 29.1175. The expected loss is 7.9078.

True energy (TeV)	0.5	1	2
Retained fraction	0.8	0.8	0.7
Lost fraction	0.2	0.2	0.3

Interpret the spectral scale

a is the unattenuated differential flux at 1 TeV. `tau` gives optical depth. The factor `exp.(-0.8 .* tau)` attenuates each energy before migration.

ANNOTATED LINES

A True and reconstructed bin indices 11–12

```
M = rowstack([[0.7, 0.2, 0.0],
              [0.1, 0.6, 0.7]])
```

Each column gives the outcome probabilities at one true energy. Each row gives contributions to one reconstructed bin. The column sums are 0.8, 0.8 and 0.7.

B Fold a differential flux 14–16

```
unit_flux = E .^ (-2) .* exp.(-0.8 .* tau)
fold(a) = M * (time .* area .* width
    .* (a .* unit_flux))
```

At each true energy, multiply the attenuated power-law flux by effective area, exposure and quadrature weight. Apply the migration matrix to the resulting vector of expected counts.

C Add the observation law 22–23

```
on ~ Poisson.(fold(a) .+ bg)
off ~ Poisson.(4.0 .* bg)
```

Add the fixed background in each bin to its reconstructed signal. The control exposure is four times the signal exposure. Conditional on the flux amplitude, the four counts are independent.

D Score one flux amplitude 25–28

```
data = record(on = [7, 8], off = [8, 12])
K = kernelof(record(on = on, off = off), a = a)
L = likelihoodof(K, data)
score = logdensityof(L, record(a = amplitude))
```

The kernel returns a named count vector for each region. At the specified amplitude the expected source counts are 16.0075 and 13.1101. The joint log likelihood is −14.91365.

Check the units

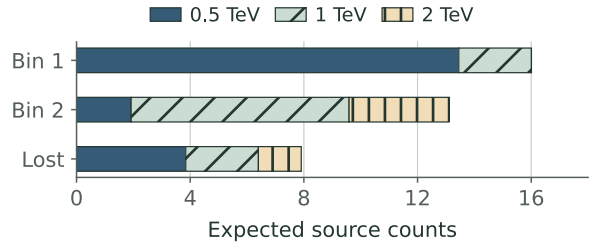
Energy is in TeV, effective area in cm², and exposure in seconds. Differential flux times an energy weight, area, and time gives a dimensionless expected count.

Follow the 0.5 TeV column

Of its 19.2158 expected counts, the response sends 13.4511 to bin 1, 1.9216 to bin 2, and 3.8432 outside both bins. Multiply the true count by 0.7, 0.1, and 0.2 respectively.

RESPONSE MATRIX AND EXPOSURE

Migration into reconstructed bins



Colour and hatching identify the true energy. The first two bars give expected source counts in the reconstructed bins. The third gives the expected loss from the corresponding column deficits.

Write the fold with named axes

```
truth = time .* area .* width .* (
    amplitude .* unit_flux)
s[.r] := M[.r, .j] * truth[.j]
```

`truth` contains the three expected counts before migration. Sum over true energy `.j` and retain reconstructed bin `.r`. The gamma-ray module implements the fold as `fold_spectrum`.

Check each detector bin

Sum each row for the expected source count. Then add the background for that detector bin.

True energy	0.5 TeV	1 TeV	2 TeV
Bin 1	13.4511	2.5564	0
Bin 2	1.9216	7.6693	3.5192

The means of the two on-source Poisson laws are 18.0075 and 16.1101. The off-source means are 8 and 12.

Detection losses

A column deficit is the probability of an event falling outside the selected bins. Renormalizing that column assigns the lost events to the measured bins.

Scale the exposure

At fixed spectrum, doubling the observing time doubles every expected source count. Changing the flux amplitude has the same linear effect on the signal.

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