Dalitz amplitudes

DENSITY IN MASS AND HELICITY ANGLE

$$\mathrm{d}P \propto mq(m)k(m)|A|^2\,\mathrm{d}m\,\mathrm{d}c$$

COMPLETE EXAMPLE

The decay $D^- \to \pi^+\pi^-\pi^-$

```
%%%
Constant-width S-wave Dalitz model.
%%%
flatppl_compat = "0.1"
mpi = 0.13957039
mD = 1.86965
mR = 0.98
width = 0.07
hep = standard_module("particle-physics", "0.1")
E(m) = (mD^2-m^2-mpi^2)/(2*m)
q(m) = (mD/m)*hep.breakup_momentum(mD, m, mpi)
k(m) = hep.breakup_momentum(m, mpi, mpi)
s12(m,c) = 2*mpi^2+m*E(m)+2*q(m)*k(m)*c

b = mR * width
bw(s) = b * hep.resonance_breitwigner(
    s, mR, width, 0.0, 0.0, 0, 1.0)
wave(s,p) = p.r*cis(p.phi)*bw(s)
amp(m,c,p) = 1 + (wave(s12(m,c),p)
    + wave(s12(m,-c),p))
separate(u,v) = 1 + abs2(u) + abs2(v)
weight(x,p) = x[1]*q(x[1])*k(x[1]) * (
    abs2(amp(x[1],x[2],p)))

# Coordinates are m(pi- pi-) and cos(theta).
support = cartprod(interval(2*mpi,mD-mpi),
    interval(-1.0,1.0))
r = elementof(interval(0.0,2.0))
phi = elementof(interval(-pi,pi))
parameters = record(r = r, phi = phi)
raw = weighted(weight, Lebesgue(support),
    parameters)
pdf = normalize(raw)
K = functionof(pdf)
event = [1.0, 0.35]
L = likelihoodof(K, event)

p0 = record(r = 0.6, phi = 0.7)
check = weight(event, p0)
```

Exchange the two negative pions

```
mirror = weight([1.0, -0.35], p0)
```

Exchanging pions 2 and 3 takes c to $-c$ and swaps s_{12} with s_{13}. Both weights round to 0.69474017. The invariants obey $s_{12} + s_{13} + m^2 = m_D^2 + 3m_\pi^2$.

ANNOTATED LINES

A Pair mass and helicity angle 9–13

```
hep = standard_module("particle-physics", "0.1")
E(m) = (mD^2-m^2-mpi^2)/(2*m)
q(m) = (mD/m)*hep.breakup_momentum(mD, m, mpi)
k(m) = hep.breakup_momentum(m, mpi, mpi)
s12(m,c) = 2*mpi^2+m*E(m)+2*q(m)*k(m)*c
```

Multiply the bachelor momentum by $\frac{m_D}{m}$ to reach the pair rest frame. There `k` is the daughter momentum. The two pairings use helicity cosines c and $-c$.

B Add amplitudes before squaring 18–21

```
wave(s,p) = p.r*cis(p.phi)*bw(s)
amp(m,c,p) = 1 + (wave(s12(m,c),p)
    + wave(s12(m,-c),p))
separate(u,v) = 1 + abs2(u) + abs2(v)
```

`amp` adds the two pion pairings to the unit reference amplitude. Line 23 then takes `abs2`. Squaring each term first gives `separate`. The difference includes interference with the reference and between the pairings.

C Declare the two couplings 28–30

```
r = elementof(interval(0.0,2.0))
phi = elementof(interval(-pi,pi))
parameters = record(r = r, phi = phi)
```

`r` sets the common magnitude and `phi` the phase in radians. Pass both to `weight` in one record. The event coordinates on lines 26–27 are the pair mass in GeV and the helicity cosine.

D Construct the event likelihood 33–36

```
pdf = normalize(raw)
K = functionof(pdf)
event = [1.0, 0.35]
L = likelihoodof(K, event)
```

The Jacobian to squared-mass coordinates is $4mqk$. `weight` uses $mqk|A|^2$ on `Lebesgue(support)`. The factor 4 cancels under `normalize`. Bind the observed `[m,c]` to the kernel in `r` and `phi`.

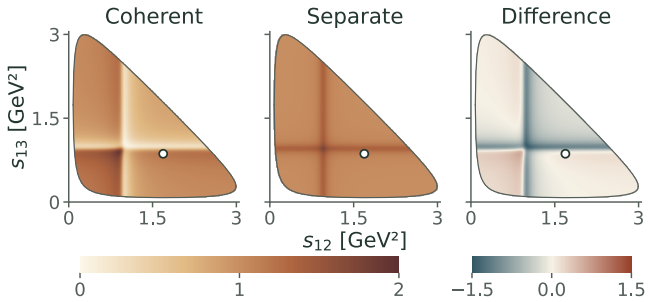
Read the marked event

```
sp = s12(event[1], event[2])
sm = s12(event[1], -event[2])
u = wave(sp, p0)
v = wave(sm, p0)
```

`abs2(1+u+v)` gives 1.1763. `separate(u,v)` gives 1.1235. Their difference, $+0.0528$, is the interference at this event.

PHASE SPACE AND INTENSITY

Interference on the Dalitz plot



At `p0`, the first two maps share an intensity scale. The third shows coherent minus separate. Red marks constructive interference and blue marks destructive interference. The circles mark `event`.

Include detector efficiency

```
eff(c) = 1 - 0.2*c^2
accepted(x,p) = eff(x[2]) * weight(x,p)
M_acc = normalize(weighted(accepted,
    Lebesgue(support), parameters))
```

This angular efficiency ranges from 0.8 to 1. Include it in the normalization integral at each coupling.

Bind several events

```
data = [[1.0, 0.35], [0.9, -0.2]]
K_events = functionof(iid(pdf, 2))
L_events = likelihoodof(K_events, data)
```

Each row is one event. For the JS engine, compute Z by external quadrature. At `p0`, $Z = 1.17878135$ gives a first-event log density of -0.52870.

Add background events

```
bg = joint(Uniform(interval(2*mpi, mD-mpi)),
    Uniform(interval(-1.0, 1.0)))
M = superpose(weighted(0.8, pdf),
    weighted(0.2, bg))
```

The uniform background contributes 20% of the events.

Amplitude and resolution assumptions

Zero daughter masses and $\ell = 0$ in `bw` give a constant width. The momenta use the pion mass. The model assumes ideal resolution. The amplitudes module gives other lineshapes and angular factors. Absolute rates require phase-space constants and a symmetry factor of $\frac{1}{2}$.

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