Rounded readings and detection flags

PROBABILITY OF A ROUNDING INTERVAL

$$P(a < Y < b) = F(b) - F(a)$$

COMPLETE EXAMPLE

Two instruments, one mean

```
%%%
Two independent reports of a shared mean.
%%%
flatppl_compat = "0.1"
meas = load_module(
    "https://raw.githubusercontent.com/
    flatppl-community/
    flatppl-stats-measurement/
    c296dc3b975be62fa759468881ee8fa568879a71/
    main.flatppl"
)
mu = elementof(reals)
lo = 0.95
hi = 1.05
cut = 1.20

p_cell = meas.gaussian_interval_probability(
    lo, hi, mu, 0.2)
p_flag = meas.gaussian_above_probability(
    cut, mu, 0.3)

cell ~ Bernoulli(p_cell)
flag ~ Bernoulli(p_flag)
reports = record(cell = cell, flag = flag)
K = kernelof(reports, mu = mu)

data = record(cell = 1, flag = 1)
L = likelihoodof(K, data)
at_one = logdensityof(L, record(mu = 1.0))
```

Check a symmetric cell

`p_at_center = 2 * invprobit(0.05 / 0.2) - 1`
The cell extends a quarter of a standard deviation on either side of the mean. Gaussian symmetry gives the same interval probability.

Check the likelihood by hand

At $\mu = 1$, the rounding probability is 0.197413 and the flag probability is 0.252493. Their product is 0.0498452, with log likelihood −2.99883.

A reading below threshold

```
other = record(cell = 1, flag = 0)
L_other = likelihoodof(K, other)
```
A zero contributes the probability of a reading at or below the threshold.

ANNOTATED LINES

A Set the reporting limits 8–11

```
mu = elementof(reals)
lo = 0.95
hi = 1.05
cut = 1.20
```
A reading rounded to 1.0 at the nearest tenth falls between 0.95 and 1.05. The second instrument records whether a separate reading exceeds 1.20. Both instruments measure the same mean.

B Evaluate the event probabilities 13–16

```
p_cell = meas.gaussian_interval_probability(
    lo, hi, mu, 0.2)
p_flag = meas.gaussian_above_probability(
    cut, mu, 0.3)
```
Pass the two edges, mean and standard deviation to the interval function. The upper-tail function takes one threshold. The two instrument scales are 0.2 and 0.3.

C Indicators for the two reports 18–21

```
cell ~ Bernoulli(p_cell)
flag ~ Bernoulli(p_flag)
reports = record(cell = cell, flag = flag)
K = kernelof(reports, mu = mu)
```
Assign one Bernoulli variable to each reported event. Conditional independence of the readings given `mu` makes their likelihood a product of two factors.

D Condition on the two reports 23–25

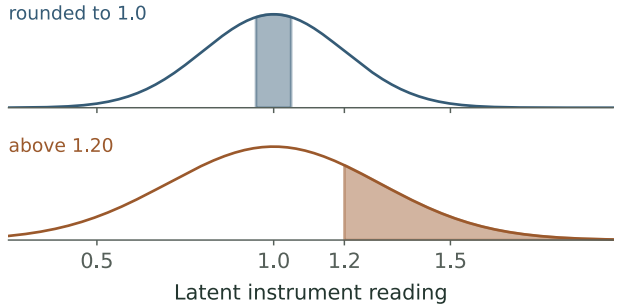
```
data = record(cell = 1, flag = 1)
L = likelihoodof(K, data)
at_one = logdensityof(L, record(mu = 1.0))
```
Set both indicator fields to one for these reports. The likelihood depends on the common mean. `at_one` evaluates it at the mean used in the figure.

Tail subtraction

When both edges exceed the mean, the measurement module subtracts upper tails. This avoids subtracting two CDF values close to one.

OBSERVATION CONVENTIONS

Rounding cell and detection threshold



At $\mu = 1$, the first instrument reports a rounded value of 1.0. The second reports that its reading exceeded 1.20. Shading shows the probability of each report.

Set the rounding cell

For a displayed value r with spacing d, use the edges $r - \frac{d}{2}$ and $r + \frac{d}{2}$. Under a continuous Gaussian law, the endpoint convention leaves the probability unchanged.

Censoring and selection

Censoring preserves the number of readings beyond a limit. For a catalogue containing only detections, model the selection probability or the detected-event count.

Two reports from one reading

The product above describes independent readings. For a rounded value and flag derived from a single reading, take the probability of the intersection.

Combine identical threshold trials

`hits ~ Binomial(20, p_flag)`
If all twenty independent readings share the same threshold and noise scale, retain their total number of detections.

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